Exercise 14

Find the differential of each function.

$$\text{(a) } y = \ln(\sin\theta) \qquad\qquad \text{(b) } y = \frac{e^x}{1 - e^x}$$

Solution

Part (a)

Compute the derivative of y.

$$\frac{dy}{d\theta} = \frac{d}{d\theta}\ln(\sin\theta)$$

$$= \frac{1}{\sin\theta} \cdot \frac{d}{d\theta}(\sin\theta)$$

$$= \frac{1}{\sin\theta} \cdot (\cos\theta)$$

$$= \cot\theta$$

Therefore, the differential of $y = \ln(\sin\theta)$ is

$$dy = \cot\theta \, d\theta.$$

Part (b)

Compute the derivative of y.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^x}{1 - e^x}\right)$$

$$= \frac{\left[\dfrac{d}{dx}(e^x)\right](1 - e^x) - \left[\dfrac{d}{dx}(1 - e^x)\right](e^x)}{(1 - e^x)^2}$$

$$= \frac{(e^x)(1 - e^x) - (-e^x)(e^x)}{(1 - e^x)^2}$$

$$= \frac{e^x}{(1 - e^x)^2}$$

Therefore, the differential of $y = e^x/(1 - e^x)$ is

$$dy = \frac{e^x}{(1 - e^x)^2}\, dx.$$